A General Coding Scheme for Two-User Fading Interference Channels

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***Abstract*— A Han-Kobayashi based achievable scheme is presented for ergodic fading two-user Gaussian interference channels (IFCs) with perfect channel state information at all nodes and Gaussian codebooks with no time-sharing. Using max-min optimization techniques, it is shown that jointly coding across all states performs at least as well as separable coding for the sub-classes of uniformly weak (every sub-channel is weak) and hybrid (mix of strong and weak sub-channels that do not achieve the interference-free sum-capacity) IFCs. For the uniformly weak IFCs, sufficient conditions are obtained for which the sum-rate is maximized when interference is ignored at both receivers.**

I. Introduction

Gaussian interference channels (IFCs) model wireless networks as a collection of two or more interfering transmit-receive pairs (links). Capacity results for two-user non-fading Gaussian IFCs are only known for specific sub-classes of IFCs such as strong [1], [2], very strong (a sub-class of strong) [3], one-sided weak [4], and very weak or noisy [5], [6], and [7]. Outer bounds for IFCs are developed in [8] and [9]. The best known inner bounds are due to Han and Kobayashi (HK) [2].

Ergodic fading and parallel Gaussian IFCs (PGICs) are IFC models that include both the fading and interference characteristics of the wireless medium. PGICs in which every sub-channel is strong and one-sided PGICs are studied in [10] and [11], respectively; both papers present achievable schemes based on coding independently for each parallel sub-channel. For PGICs, [12] determines the conditions on the channel coefficients and power constraints for which independent transmission across sub-channels and treating interference as noise is optimal.

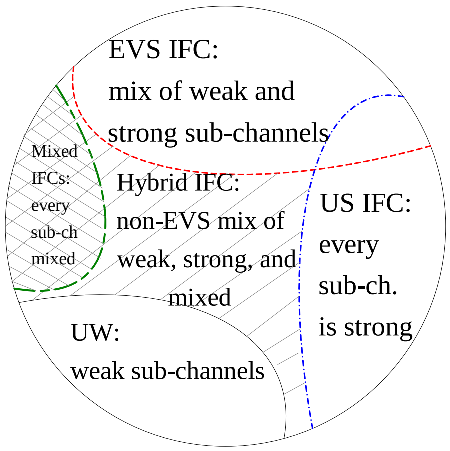
For ergodic fading Gaussian IFCs, henceforth referred to simply as IFCs, we developed the sum-capacity and separability for specific sub-classes in [13] and [14]. In contrast to the non-fading case, we proved that ergodic fading IFCs with a mix of weak and strong sub-channels that satisfy a specific set of conditions can achieve the sum of the interference-free capacities of the two intended links; we identified such channels as the sub-class of *ergodic very strong* (EVS) IFCs. For this sub-class, we showed that jointly coding across all sub-channels (i.e., transmitting the same message in every sub-channel) and requiring the receivers to decode the transmissions from both users achieves the capacity region. Furthermore, in [13], we outlined the optimality of this achievable coding scheme for a sub-class of *uniformly strong* (US) IFCs in which every sub-channel is strong. The US and EVS sub-classes overlap but in general are not the same (see Fig. 1). For one-sided *uniformly weak* (UW) IFCs, in which every sub-channel is weak, we conjectured the optimality of ignoring interference and separable coding in [14]. Converse proofs for each of the above-mentioned sub-channels as well as for a sub-class of *uniformly mixed* two-sided IFCs, comprised of two complementary UW and US one-sided IFCs, is developed in [15].

For one-sided and two-sided IFCs, [15] identifies a sub-class of *hybrid* IFCs (which is complementary to all previously identified sub-classes; see Fig. 1) comprised of a mix of strong and weak sub-channels or a mix of strong, weak, and mixed sub-channels, respectively, for which the EVS conditions are not satisfied. Specifically, for one-sided IFCs, [15] unifies the above-mentioned capacity results using a HK-based achievable scheme that uses joint coding and no time-sharing such that across all sub-channels, the interfering transmitter sends a common and a private message. For the hybrid sub-class, this HK-based scheme is shown to achieve a sum-rate at least as large as that achieved by separable coding in which only common and private messages are sent in the strong and weak sub-channels, respectively.

In this paper, we develop the HK-based achievable scheme using joint coding for two-sided IFCs. We demonstrate the optimality of transmitting only common messages for the two-sided EVS and US IFCs. The sum-capacity of two-sided UW IFCs remains open; for the proposed HK-based joint coding scheme we determine a set of sufficient conditions for which ignoring interference at both receivers and separable coding maximize the sum-rate. Finally, we show that in general for both the two-sided weak and the hybrid sub-classes, joint coding of both private and common messages across all sub-channels achieves at least as large a sum-rate as separable coding. Two-sided ergodic fading Gaussian IFCs are studied in [16] using a simplified form of the HK region to determine the power policies that maximize a sum-rate inner bound. In contrast, we focus on the problem of separability and use a max-min optimization technique to unify known and new results for all sub-classes. The paper is organized as follows. In Section II, we present the channel models studied. In Section III, we summarize our main results. We conclude in Section IV.

II. Channel Model

A two-user ergodic fading Gaussian IFC consists of two transmit-receive pairs, each pair indexed by k, for $k = 1, 2$.



Two-user Ergodic Fading Two-sided IFCs

Fig. 1. A Venn diagram representation of the four sub-classes of ergodic fading IFCs.

In each use of the channel, transmitter k transmits the signal X_k while receiver k receives Y_k, $k \in \mathcal{K}$. For $\mathbf{X} = [X_1 \; X_2]^T$, the channel output vector $\mathbf{Y} = [Y_1 \; Y_2]^T$ is given by

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z} \tag{1}$$

where $\mathbf{Z} = [Z_1 \; Z_2]^T$ is a noise vector with entries that are zero-mean, unit variance, circularly symmetric complex Gaussian noise variables and $\mathbf{H}$ is a random matrix of fading gains with entries $H_{m,k}$, for all $m, k \in \{1, 2\}$, such that $H_{m,k}$ denotes the fading gain between receiver m and transmitter k. We assume the fading process $\{\mathbf{H}\}$ is stationary and ergodic but not necessarily Gaussian. Note that the channel gains $H_{m,k}$, for all m and k, are not assumed to be independent; however, $\mathbf{H}$ is known instantaneously at all the transmitters and receivers. A one-sided fading Gaussian IFC results when either $H_{1,2} = 0$ or $H_{2,1} = 0$. A two-sided IFC can be viewed as a collection of two complementary one-sided IFCs, one with $H_{1,2} = 0$ and the other with $H_{2,1} = 0$.

Over n uses of the channel, the transmit sequences $\{X_{k,i}\}$ are constrained in power according to

$$\sum_{i=1}^{n} |X_{k,i}|^2 \leq n\overline{P}_k \;, \text{ for all } k = 1, 2. \tag{2}$$

Since the transmitters know the fading states of the links on which they transmit, they can allocate their transmitted signal powers according to the channel state information. We write $\underline{P}(\mathbf{H})$ with entries $P_k(\mathbf{H})$ for all k to explicitly describe the power policy for the entire set of random fading states. For an ergodic fading channel, (2) then simplifies to

$$\mathbb{E}\left[P_k(\mathbf{H})\right] \leq \overline{P}_k \quad \text{for all } k = 1, 2, \tag{3}$$

where the expectation in (3) is over the distribution of $\mathbf{H}$. We denote the set of all feasible policies $\underline{P}\left(\mathbf{h}\right)$, i.e., the power policies whose entries satisfy (3), by $\mathcal{P}$.

Our definitions of average error probabilities, capacity regions, and achievable rate pairs (R_1, R_2) for the IFC mirror the standard information-theoretic definitions. Throughout the sequel, we use the terms fading states and sub-channels interchangeably and refer to the ergodic fading IFC as simply IFC. $\mathbb{E}\left[\cdot\right]$ denotes expectation and $C(x)$ denotes $\log(1 + x)$ where the logarithm is to the base 2.

III. Achievable Scheme

We consider an HK-based achievable scheme using Gaussian codebooks without time-sharing and joint encoding and decoding across all sub-channels. We seek to determine the power fractions allocated to private and common messages at each transmitter that maximizes the sum-rate. Our motivation stems from the fact that joint coding is optimal for EVS and US IFCs [13] and achieves at least as large a sum-rate as separable coding for hybrid one-sided IFCs [15]. We outline the achievable scheme below.

Thus, transmitter k transmits the same message pair (w_{kc}, w_{kp}) in every sub-channel where w_{kc} and w_{kp} are the common and private messages respectively. Each receiver decodes by jointly decoding using the received signals from all sub-channels. Let $\alpha_{k,\mathbf{H}} \in [0, 1]$ and $\overline{\alpha}_{k,\mathbf{H}} = 1 - \alpha_{k,\mathbf{H}}$ denote the power fractions at transmitter k allocated to transmitting the private and common messages, respectively, in sub-channel $\mathbf{H}$. The two transmitted signals in each use of sub-channel $\mathbf{H}$ are

$$X_1\left(\mathbf{H}\right) = \sqrt{\alpha_{1,\mathbf{H}} P_1\left(\mathbf{H}\right)} V_{1\mathbf{H}} + \sqrt{\overline{\alpha}_{1,\mathbf{H}} P_1\left(\mathbf{H}\right)} U_{1\mathbf{H}} \tag{4a}$$

$$X_2\left(\mathbf{H}\right) = \sqrt{\alpha_{2,\mathbf{H}} P_2\left(\mathbf{H}\right)} V_{2\mathbf{H}} + \sqrt{\overline{\alpha}_{2,\mathbf{H}} P_2\left(\mathbf{H}\right)} U_{2\mathbf{H}} \tag{4b}$$

where $V_{k\mathbf{H}}$ and $U_{k\mathbf{H}}$, $k = 1, 2$, are independent zero-mean unit variance Gaussian random variables, for all $\mathbf{H}$. We use the notation $V_{k\mathbf{H}}$ and $U_{k\mathbf{H}}$ to indicate that the random variables are mutually independent for every instantiation of $\mathbf{H}$, i.e., independent codebooks in each sub-channel. Let $\underline{\alpha}_{\mathbf{H}}$ denote a vector of power fractions with entries $\alpha_{k,\mathbf{H}}$, $k = 1, 2$.

In [17, Theorem 4], the authors present the HK region achieved by superposition coding. Assuming no time-sharing, for the Gaussian signaling in (4) and with joint coding, one can directly extend the analysis in [17, Theorem 4] to ergodic fading Gaussian IFCs. The following Proposition based on [17, Theorem 4] summarizes the resulting rate bounds.

Proposition 1: A rate pair (R_1, R_2) is achievable for a HK scheme with superposition coding and no time-sharing for ergodic fading IFCs if

$$R_k \leq B_k\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}\left(\mathbf{H}\right)\right) \tag{5}$$

$$R_1 + R_2 \leq B_3\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}\left(\mathbf{H}\right)\right) \tag{6}$$

$$R_1 + R_2 \leq B_4\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}\left(\mathbf{H}\right)\right) \tag{7}$$

$$R_1 + R_2 \leq B_5\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}\left(\mathbf{H}\right)\right) \tag{8}$$

$$2R_1 + R_2 \leq B_6\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}\left(\mathbf{H}\right)\right) \tag{9}$$

$$R_1 + 2R_2 \leq B_7\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}\left(\mathbf{H}\right)\right) \tag{10}$$

where

$$B_k = \mathbb{E}\left[C\left(\frac{|H_{k,k}|^2 P_k(\mathbf{H})}{1+\alpha_{j,\mathbf{H}}|H_{k,j}|^2 P_j(\mathbf{H})}\right)\right], \quad j, k = 1, 2, j \neq k \tag{11}$$

$$B_3 = \mathbb{E}\left[C\left(\frac{|H_{1,1}|^2 P_1(\mathbf{H}) + |H_{1,2}|^2 \overline{\alpha}_{2,\mathbf{H}} P_2(\mathbf{H})}{1+\alpha_{2,\mathbf{H}}|H_{1,2}|^2 P_2(\mathbf{H})}\right)\right] \tag{12}$$

$$+\mathbb{E}\left[C\left(\frac{|H_{2,2}|^2 \alpha_{2,\mathbf{H}} P_2(\mathbf{H})}{1+\alpha_{1,\mathbf{H}}|H_{2,1}|^2 P_1(\mathbf{H})}\right)\right]$$

$$B_4 = B_3|_{\text{indices 1 and 2 swapped}} \tag{13}$$

$$B_5 = \mathbb{E}\left[C\left(\frac{\alpha_{1,\mathbf{H}}|H_{1,1}|^2 P_1(\mathbf{H}) + |H_{1,2}|^2 \overline{\alpha}_{2,\mathbf{H}} P_2(\mathbf{H})}{1+\alpha_{2,\mathbf{H}}|H_{1,2}|^2 P_2(\mathbf{H})}\right)\right] \tag{14}$$

$$+\mathbb{E}\left[C\left(\frac{\alpha_{2,\mathbf{H}}|H_{2,2}|^2 P_2(\mathbf{H}) + |H_{2,1}|^2 \overline{\alpha}_{1,\mathbf{H}} P_1(\mathbf{H})}{1+\alpha_{1,\mathbf{H}}|H_{2,1}|^2 P_1(\mathbf{H})}\right)\right]$$

$$B_6 = \mathbb{E}\left[C\left(\frac{|H_{1,1}|^2 P_1(\mathbf{H}) + |H_{1,2}|^2 \overline{\alpha}_{2,\mathbf{H}} P_2(\mathbf{H})}{1+\alpha_{2,\mathbf{H}}|H_{1,2}|^2 P_2(\mathbf{H})}\right)\right] \tag{15}$$

$$+\mathbb{E}\left[C\left(\frac{|H_{1,1}|^2 \alpha_{1,\mathbf{H}} P_1(\mathbf{H})}{1+\alpha_{2,\mathbf{H}}|H_{1,2}|^2 P_2(\mathbf{H})}\right)\right]$$

$$+\mathbb{E}\left[C\left(\frac{\alpha_{2,\mathbf{H}}|H_{2,2}|^2 P_1(\mathbf{H}) + |H_{2,1}|^2 \overline{\alpha}_{1,\mathbf{H}} P_1(\mathbf{H})}{1+\alpha_{1,\mathbf{H}}|H_{2,1}|^2 P_1(\mathbf{H})}\right)\right]$$

$$B_7 = B_6|_{\text{indices 1 and 2 swapped}} \tag{16}$$

Theorem 2: The sum-capacity of ergodic fading IFCs is lower bounded by

$$\max_{\underline{P}(\mathbf{H})\in\mathcal{P},\alpha_{k,\mathbf{H}}\in[0,1]} \min_{m\in\{1,2,3,4,5,6\}} S_m\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) \tag{17}$$

where

$$S_1\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) = B_1\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) + B_2\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right), \tag{18}$$

$$S_j\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) = B_{j+1}\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right),\ j=2,3,4 \tag{19}$$

$$S_5\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) = \left(B_6\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) + B_2\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right)\right)/2 \tag{20}$$

$$S_6\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) = \left(B_7\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) + B_1\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right)\right)/2. \tag{21}$$

(a) For EVS IFCs, the sum-capacity $S_1\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right)$ is achieved by choosing $\underline{\alpha}^*_{\mathbf{H}} = \underline{0}$ for all $\mathbf{H}$ and $\underline{P}^*(\mathbf{H}) = \underline{P}^{(wf)}(\mathbf{H})$ provided $S_1\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right) < S_j\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right)$, for all $j>1$, where $\underline{P}^{(wf)}(\mathbf{H})$ is the optimal waterfilling policy for the two interference-free direct links. *(b)* For US IFCs, the sum-capacity is achieved by $\underline{\alpha}^*_{\mathbf{H}} = 0$, for all $\mathbf{H}$ and is given by

$$\max_{\underline{P}(\mathbf{H})\in\mathcal{P}} \min_{m\in\{1,2,3\}} S_m\left(\underline{0}, \underline{P}(\mathbf{H})\right). \tag{22}$$

(c) For UM IFCs, the sum-capacity is achieved by choosing $\alpha^*_{k,\mathbf{H}} = 1$ and $\alpha^*_{j,\mathbf{H}} = 0$, $j \neq k$, where k and j are the receivers that see weak and strong interference, respectively, and is given by

$$\max_{\underline{P}(\mathbf{H})\in\mathcal{P}} \min_{m\in\{2,3\}} S_m\left(\underline{\alpha}^*_{\mathbf{H}}, \underline{P}(\mathbf{H})\right). \tag{23}$$

(d) For UW IFCs, the sum-rate is maximized by $\underline{\alpha}^*_{\mathbf{H}} = \underline{1}$ if, for every $\underline{P}(\mathbf{H}) \in \mathcal{P}$,

$$|H_{2,2}|^2 > \left(1+|H_{2,1}|^2 P_1(\mathbf{H})\right)|H_{1,2}|^2 \tag{24}$$

$$|H_{1,1}|^2 > \left(1+|H_{1,2}|^2 P_2(\mathbf{H})\right)|H_{2,1}|^2 \tag{25}$$

and is given by

$$\max_{\underline{P}(\mathbf{H})\in\mathcal{P}} S_1\left(\underline{1}, \underline{P}(\mathbf{H})\right). \tag{26}$$

For a hybrid one-sided IFC, the achievable sum-rate is maximized by

$$\alpha^*_{k,\mathbf{H}} = \begin{cases} \alpha_k(\mathbf{H}) \in (0,1] & \mathbf{H} \text{ is weak} \\ 0 & \mathbf{H} \text{ is strong.} \end{cases}, k=1,2, \tag{27}$$

and is given by (17) for this choice of $\underline{\alpha}^*_{\mathbf{H}}$.

Remark 3: The conditions in (24) and (25) hold for all feasible power policies $\underline{P}(\mathbf{H})$, i.e., policies satisfying the fading averaged constraint in (3), and thus, are quite restrictive in defining the set of channel gains for which ignoring interference is optimal for UW IFCs. However, the analysis and the conditions (24) and (25) also hold for ergodic channels with a per-symbol or equivalently per-fading state power constraint for which determining the largest values of the right-side of (24) and (25) is relatively easier.

Proof: Our proof relies on using the fact that the maximization of the minimum of two functions, say $f_1\left(\alpha_{\mathbf{H}}, \underline{P}(\mathbf{H})\right)$ and $f_2\left(\alpha_{\mathbf{H}}, \underline{P}(\mathbf{H})\right)$ is equivalent to a *minimax* optimization problem (see for e.g., [18, II.C]) for which the maximum sum-rate S^* is given by the following three cases. In each case, the optimal $\underline{P}^*(\mathbf{H})$ and $\alpha^*_{\mathbf{H}}$ maximize the smaller of the two functions and therefore maximize both for the case when the two functions are equal. The three cases are

$$\text{Case } 1: \quad S^* = f_1\left(\alpha^*_{\mathbf{H}}, \underline{P}^*(\mathbf{H})\right) < f_2\left(\alpha^*_{\mathbf{H}}, \underline{P}^*(\mathbf{H})\right) \tag{28a}$$

$$\text{Case } 2: \quad S^* = f_2\left(\alpha^*_{\mathbf{H}}, \underline{P}^*(\mathbf{H})\right) < f_1\left(\alpha^*_{\mathbf{H}}, \underline{P}^*(\mathbf{H})\right) \tag{28b}$$

$$\text{Case } 3: \quad S^* = f_1\left(\alpha^*_{\mathbf{H}}, \underline{P}^*(\mathbf{H})\right) = f_2\left(\alpha^*_{\mathbf{H}}, \underline{P}^*(\mathbf{H})\right) \tag{28c}$$

From (17), the sum-rate is the solution to a max-min optimization of $f_1(\cdot) = S_1(\cdot)$ and $f_2(\cdot) = \min_{j>1} S_j(\cdot)$. We now consider each sub-class separately.

Ergodic very strong: By definition, an EVS IFC results when the sum of the interference-free capacities of the two links can be achieved. From (28), one special case of the max-min optimization in (17) corresponds to the EVS sub-class. This results when

$$\max_{\underline{P}(\mathbf{H}),\alpha_{\mathbf{H}}} S_1\left(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})\right) = S_1\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right)$$

$$< S_j\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right), \text{ for all } j>1, \tag{29}$$

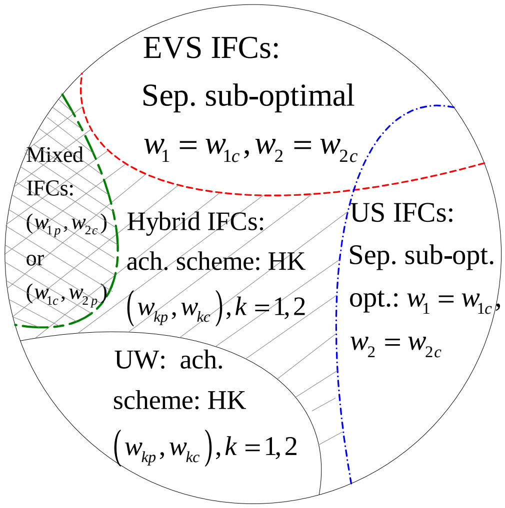


Fig. 2. Two-sided ergodic fading IFCs: overview of known results.

where we have used the fact that the ergodic capacities of the two interference-free links are maximized by the optimal single-user waterfilling policies [19], denoted by $\underline{P}^{(wf)}(\mathbf{H})$ with entries $P_k^{(wf)}(H_{k,k})$. Note that $S_2(\underline{0}, \underline{P}(\mathbf{H}))$ and $S_3(\underline{0}, \underline{P}(\mathbf{H}))$ are the multiple-access sum-capacities at receivers 1 and 2, respectively, such that

$$S_2(\underline{0}, \underline{P}(\mathbf{H})) = S_3(\underline{0}, \underline{P}(\mathbf{H}))|_{\text{swap indices 1 and 2}}. \tag{30}$$

We now show that (29) simplifies to the requirement

$$S_1\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right) < \min\left(S_2\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right), S_3\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right)\right) \tag{31}$$

The proof follows trivially by expanding the terms $S_j\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right)$ for all $j > 1$ and comparing them to $S_1\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right)$. We illustrate this for $S_4(\underline{0}, \underline{P}(\mathbf{H}))$ as follows.

$$\begin{aligned} S_4(\underline{0}, \underline{P}(\mathbf{H})) &= \mathbb{E}\left[C\left(|H_{1,2}|^2 P_2^{(wf)}(\mathbf{H})\right)\right] && \text{(32a)} \\ &\quad + \mathbb{E}\left[C\left(|H_{2,1}|^2 P_1^{(wf)}(\mathbf{H})\right)\right] \\ &> \mathbb{E}\left[C\left(|H_{2,2}|^2 P_2^{(wf)}(\mathbf{H})\right)\right] && \text{(32b)} \\ &\quad + \mathbb{E}\left[C\left(|H_{2,1}|^2 P_1^{(wf)}(\mathbf{H})\right)\right] \\ &\geq S_3\left(\underline{0}, \underline{P}^{(wf)}(\mathbf{H})\right) && \text{(32c)} \end{aligned}$$

where (32b) follows from simplifying (31) by expanding the multiple-access sum-capacity terms and (32b) follows from using chain rule for mutual information. We note that (31) is the EVS condition developed in [13, Th. 7] (see also [15, Theorem 2]). The sum-capacity follows from noting that the sum of the capacities of two interference-free links is an outer bound on the IFC sum-capacity.

Uniformly strong: One can verify in a straightforward manner that $S_1(\alpha_{\mathbf{H}}, \underline{P}(\mathbf{H}))$ is maximized $\alpha_{1,\mathbf{H}}^* = \alpha_{2,\mathbf{H}}^* = 0$. Furthermore, when all sub-channels are strong, i.e., when $\Pr[|H_{1,2}| > |H_{2,2}|] = 1$, the bound $S_2(\alpha_{\mathbf{H}}, \underline{P}(\mathbf{H}))$ in (19) can be rewritten as

$$\begin{aligned} &\mathbb{E}\left[C\left(|H_{1,1}|^2 P_1(\mathbf{H}) + |H_{1,2}|^2 P_2(\mathbf{H})\right)\right] \\ &\quad - \mathbb{E}\left[C\left(1 + \alpha_{2,\mathbf{H}} |H_{1,2}|^2 P_2(\mathbf{H})\right)\right] \\ &\quad + \mathbb{E}\left[C\left(\frac{|H_{2,2}|^2 \alpha_{2,\mathbf{H}} P_2(\mathbf{H})}{1 + \alpha_{1,\mathbf{H}} |H_{2,1}|^2 P_1(\mathbf{H})}\right)\right]. \end{aligned} \tag{33}$$

Using the US condition, one can verify that for every choice of $\underline{P}(\mathbf{H})$, $S_2(\alpha_{\mathbf{H}}, \underline{P}(\mathbf{H}))$ is maximized by $\alpha_{1,\mathbf{H}}^* = \alpha_{2,\mathbf{H}}^* = 0$, i.e., $w_k = w_{k,c}$, $k = 1, 2$. Since $S_3(\cdot)$ is obtained from $S_2(\cdot)$ by swapping the indices, the above choice also maximizes $S_3(\cdot)$. Transmitting only common messages at both transmitters results in multiple-access regions at both receivers; one can use the properties of these multiple-access regions to show that the remaining sum-rate bounds are at least as much as the minimum of $S_j(\underline{0}, \cdot)$, $j = 1, 2, 3$, such that the maximum achievable sum-rate is given by (22). The outer bound analysis in [15, Theorem 3] helps establish that (22) is the US sum-capacity.

Uniformly mixed: Without loss of generality, assume $\Pr[|H_{2,1}|^2 > |H_{1,1}|^2] = 1$ and $\Pr[|H_{1,2}|^2 < |H_{2,2}|^2] = 1$, i.e., receivers 1 and 2 experience weak and strong interference, respectively. Comparing with the US case, we choose $\alpha_{1,\mathbf{H}}^* = 0$. Furthermore, is is straightforward to verify that $S_2(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H}))$ is maximized by $\alpha_{2,\mathbf{H}}^* = 1$ while S_3 is independent of $\alpha_{1,\mathbf{H}}$ for $\alpha_{1,\mathbf{H}}^* = 0$. For $j \geq 4$, $S_j\left(\alpha_{1,\mathbf{H}}^* = 0, \alpha_{2,\mathbf{H}}, \underline{P}(\mathbf{H})\right)$ is in general maximized by a $\alpha_{2,\mathbf{H}} \neq 1$. Evaluating all functions at $\left(\alpha_{1,\mathbf{H}}^*, \alpha_{2,\mathbf{H}}^*\right) = (0, 1)$ and for any $\underline{P}(\mathbf{H})$, one can verify that $S_1(\cdot) = S_2(\cdot)$, $S_4(\cdot) = S_3(\cdot)$, and $S_j(\cdot) > \min(S_2(\cdot), S_3(\cdot))$, $j = 5, 6$. Thus, the max-min optimization simplifies to (23). Finally, using outer bounds developed for two complementary one-sided IFCs (see [15, Theorem 5]), we can show that (23) is the sum-capacity.

Uniformly weak: For this sub-class of channels, it is straightforward to see that the conditions for Case 1 in (29) will not be satisfied (as otherwise the sub-class would be EVS), and thus, $\alpha_{k,\mathbf{H}}^* \neq 0$ for $k = 1, 2$. For the case with one-sided interference, in [15, Th. 4], we show that transmitting only private messages at the interfering transmitter maximizes the sum-rate and is in fact sum-capacity optimal. However, for the two-sided case, the choice of $\alpha_{k,\mathbf{H}}^* = 1$ for all k, i.e., $w_k = w_{k,p}$ for all k, does not necessarily maximize the sum-rate. Consider the function $S_2(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H}))$ in (19). From (12), it can be rewritten as

$$\begin{aligned} S_2(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H})) &= \mathbb{E}\Big[C\left(|H_{1,1}|^2 P_1(\mathbf{H}) + |H_{1,2}|^2 P_2(\mathbf{H})\right) \\ &\quad + C\left(|H_{2,1}|^2 \alpha_{1,\mathbf{H}} P_1(\mathbf{H}) + |H_{2,2}|^2 \alpha_{2,\mathbf{H}} P_2(\mathbf{H})\right) \\ &\quad - C\left(1 + \alpha_{2,\mathbf{H}} |H_{1,2}|^2 P_2(\mathbf{H})\right) - C\left(1 + \alpha_{1,\mathbf{H}} |H_{2,1}|^2 P_1(\mathbf{H})\right)\Big]. \end{aligned} \tag{34}$$

For $\alpha_{1,\mathbf{H}} = 0$, one can use the concavity of the logarithm to show that $S_2(\underline{\alpha}_{\mathbf{H}}, \underline{P}(\mathbf{H}))$ is maximized by $\alpha^*_{2,\mathbf{H}} = 1$; however, for any $\alpha_{1,\mathbf{H}} > 0$ and a $P_1(\mathbf{H}) \neq 0$, $S_2(\cdot)$ may not be maximized by $\alpha^*_{2,\mathbf{H}} = 1$. Rewriting the second and third term in (34) as

$$\mathbb{E}\left[C\left(|H_{2,1}|^2 \alpha_{1,\mathbf{H}} P_1(\mathbf{H})\right) + C\left(\frac{|H_{2,2}|^2 \alpha_{2,\mathbf{H}} P_2(\mathbf{H})}{1 + |H_{2,1}|^2 \alpha_{1,\mathbf{H}} P_1(\mathbf{H})}\right) - C\left(1 + \alpha_{2,\mathbf{H}} |H_{1,2}|^2 P_2(\mathbf{H})\right)\right]$$

we see that $\underline{\alpha}^*_{\mathbf{H}} = \underline{1}$ maximizes S_2 only if (24) is satisfied for all $\underline{P}(\mathbf{H})$. One can similarly show that the choice $\underline{\alpha}^*_{\mathbf{H}} = \underline{1}$ maximizes S_3 if (25) is satisfied for all $\underline{P}(\mathbf{H})$. Both (24) and (25) need to be satisfied for $\underline{\alpha}_{\mathbf{H}} = \underline{1}$ to maximize S_4. In general, (24) and (25) do not guarantee that $\underline{\alpha}^*_{\mathbf{H}} = \underline{1}$ maximizes S_1, S_5, and S_6; however, since $S_j(\underline{\alpha}^*_{\mathbf{H}} = \underline{1}, \underline{P}_{\mathbf{H}}) = S_2(\underline{\alpha}^*_{\mathbf{H}} = \underline{1}, \underline{P}_{\mathbf{H}})$, $j = 1, 5, 6$, the maximum sum-rate for the UW sub-class is given by (26) when (24) and (25) hold.

Hybrid: This sub-class includes all IFCs with a mix of weak, strong, and mixed sub-channels that to do not satisfy the EVS condition. Let $\underline{\alpha}^{(s)}_{\mathbf{H}}$, $\underline{\alpha}^{(m)}_{\mathbf{H}}$, and $\underline{\alpha}^{(w)}_{\mathbf{H}}$ denote the vector of power fractions for the private messages in the strong, mixed, and weak sub-channels, respectively. Using the linearity of expectation, we can write the expressions for $S_j(\cdot)$ for all j as sums of expectations of the appropriate bounds over the collection of strong, mixed, and weak sub-channels. Let $S^{(s)}_j(\cdot)$ $S^{(m)}_j(\cdot)$, and $S^{(w)}_j(\cdot)$ denote the expectation over the strong, mixed, and weak sub-channels, respectively, such that $S_j(\cdot) = S^{(s)}_j(\cdot) + S^{(m)}_j + S^{(w)}_j(\cdot)$, for all j. Let $\underline{\alpha}^{(s)}_{\mathbf{H}}$, $\underline{\alpha}^{(m)}_{\mathbf{H}}$, and $\underline{\alpha}^{(w)}_{\mathbf{H}}$ denote the optimal $\underline{\alpha}^*_{\mathbf{H}}$ for the weak and strong states, respectively. For those sub-channels which are strong and mixed, one can use the arguments above to show that $\underline{\alpha}^{(s)}_{\mathbf{H}} = 0$ and $\underline{\alpha}^{(m)}_{\mathbf{H}} = (0, 1)$ maximize $S^{(s)}_j(\cdot)$ and $S^{(m)}_j$, respectively. For the weak sub-channels, as for the UW sub-class, the entries of the optimal $\underline{\alpha}^{(w)}_{\mathbf{H}}$ can take on any value in the range $(0, 1]$. ■

IV. Concluding Remarks

We have presented a Han-Kobayashi based achievable scheme for two-sided ergodic fading Gaussian IFCs. Relying on converse results in [15], we have shown the optimality of transmitting only common messages for the sub-classes of EVS and US IFCs. For the hybrid sub-classes, we have shown that the proposed joint coding scheme does at least as well as separable coding by exploiting the strong sub-channels to reduce interference in the weak sub-channels. In contrast to the one-sided UW sub-class for which ignoring interference and separable coding are optimal, we have argued here that in general joint coding is required for the two-sided UW sub-class. While the sum-capacity optimal scheme is unknown for this sub-class, we have developed a set of sufficient conditions under which ignoring interference and separable coding are optimal. Our results are summarized by the Venn diagrams in Fig. 2.

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